Random advection in a fractal medium with finite correlation length

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We investigate tracer advection in fractal media with finite correlation length. The process is superdiffusive at early times and transforms to a classical diffusion regime later. At large time the spatial tail of the concentration has a two-stage structure with a long-distant part corresponding to superdiffusive regime.

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I. INTRODUCTION

Tracer advection in random velocity field with long-range correlations remains a subject of active interest (see Refs. [1–5], and references therein). Such velocity distribution may describe fluid flow in media with fractal properties, e.g., moisture infiltration in the rock matrix over the fracture set with fractal structure [2,6,7]. The tracer transport in a self-similar velocity field (characterized by power-law decay of pair correlation function) was studied in Refs. [8,9]. It has been shown that the transport is superdiffusive when the power exponent of the pair correlation function is less than 2. Along with a temporal dependence of characteristic size of the main body of a tracer particle cloud $R(t)$ examined in Refs. [8,9], the behavior of the concentration at large distances $r \gg R(t)$ (in the tail) may be very important (especially, for the reliability assessments of radioactive waste disposal in geological media). An expression for the concentration tail in the stretched Gaussian form was proposed in Refs. [10–12] for tracer subdiffusion over fractal basing on scale arguments. It has been shown in Ref. [9] that in the problem of random advection the spatial dependence of concentration in the tail is described by a "compressed" Gaussian form depending on a self-similar variable, which is the same as for the main body of concentration.

It should be noted that in Ref. [8] as well as in Ref. [9] the spatial interval of self-similarity of the advection-velocity field (where its correlation function has power-law decay) was supposed to extend up to infinity. In reality, this interval may be bounded from above resulting from a finite correlation length of fractal medium (e.g., in percolation media above the percolation threshold). Note that in the problem of moisture infiltration over the fracture system, finiteness of the correlation length leads to the appearance of mean infiltration velocity [2].

The aim of the present paper is to study the influence of a finite correlation length on tracer transport over fractal medium in the random advection model. Special attention is paid to the analysis of concentration behavior at large distances (concentration tails).

II. PROBLEM FORMULATION

The advection in a given stationary velocity field $\vec{v}(\vec{r})$ is described by the following equation for the particles' concentration $c(\vec{r},t)$:

$$\frac{\partial c}{\partial t} + \nabla(\vec{v}c) = 0. \tag{1}$$

The velocity field in Eq. (1) satisfies incompressibility equation div $\vec{v}(\vec{r})=0$. We consider the problem with initial condition $c(\vec{r},0)=c_0(\vec{r})$.

The advection velocity may be represented in the form

$$\vec{v}(\vec{r}) = \vec{u} + \vec{V}(\vec{r}), \tag{2}$$

where

$$\langle \vec{v}(\vec{r})\rangle = \vec{u}, \quad \langle \vec{V}(\vec{r})\rangle = 0, \tag{3}$$

$\vec{u}$ is the mean velocity, and $\langle\cdots\rangle$ means averaging over ensemble of realization, or spatial averaging over the scales larger than correlation length ξ.

In the spatial interval of fractality the pair correlation function of the fluctuating velocity components obey power law

$$K_{ij}^{(2)}(\vec{r}_1,\vec{r}_2) = \langle V_i(\vec{r}_1)V_j(\vec{r}_2)\rangle \propto V^2\left(\frac{a}{r}\right)^{2h}, \quad a \ll r < \xi, \tag{4}$$

where $r=|\vec{r}_1-\vec{r}_2|$, a is a lower bound of the fractality interval, and V^2 determines the characteristic value of $K_{ij}^{(2)}$ at $r \lessdot a$. Outside the fractality interval, at the distances $r \gg \xi$, correlations decay exponentially fast.

With considering these properties, the Fourier transform of the two-point correlation function has the form

$$K_{ij}^{(2)}\{\vec{k}_1,\vec{k}_2,\xi\} = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)K_{ij}^{(2)}\{\vec{k}_1,\xi\}, \tag{5}$$

$$K_{ij}^{(2)}\{\vec{k},\xi\} \propto V^2a^{2h}\begin{cases} k^{2h-3}, & k \gg \xi^{-1}, \\ \xi^{3-2h}, & k \ll \xi^{-1}, \end{cases} \tag{6}$$

where $k=|\vec{k}|$. From here, one can see that $K_{ij}^{(2)}\{\vec{k},\xi\}$ is a homogenous function of variables $\vec{k}$ and ξ^{-1}, that is,

$$K_{ij}^{(2)}\{\lambda\vec{k},\xi/\lambda\} = \lambda^{2h-3}K_{ij}^{(2)}\{\vec{k},\xi\}. \tag{7}$$

By analogy with the theory of critical phenomena [18], we call the parameter h the scaling dimension of velocity fluctuation $\vec{V}(\vec{r})$. Similar relations are valid for any multipoint velocity correlator.

According to Refs. [8,9], anomalous diffusion regimes in random advection model (under the condition $\xi\to\infty$) occur only when the exponent in Eq. (4) is less than 2, i.e., $h<1$. Henceforth we consider only this nontrivial case.

III. GREEN'S FUNCTION AND SCALING ANALYSIS

Tracer concentration at an arbitrary instant can be expressed through its initial distribution by

$$c(\vec{r},t) = \int d^3r' G(\vec{r},\vec{r}\,';t)c_0(\vec{r}\,'), \tag{8}$$

where $G(\vec{r},\vec{r}\,';t)$ is the Green's function being the solution of the equation

$$\left\{\frac{\partial}{\partial t} + \frac{\partial}{\partial x_i}v_i(\vec{r})\right\}G(\vec{r},\vec{r}\,';t) = 0 \tag{9}$$

with the initial condition

$$G(\vec{r},\vec{r}\,',0) = \delta(\vec{r}-\vec{r}\,'). \tag{10}$$

Of practical interest is the concentration averaged over an ensemble of medium realizations $\bar{c}(\vec{r},t) \equiv \langle c(\bar{r},t)\rangle$. It satisfies the equation obtained from Eq. (8) by replacing G with $\bar{G}$, where $\bar{G}(\vec{r}-\vec{r}\,',t) \equiv \langle G(\vec{r},\vec{r}\,';t)\rangle$ is the ensemble-averaged Green's function (henceforth called Green's function for simplicity). We stress that the ensemble-averaged functions $\bar{c}$ and $\bar{G}$ refer to the whole space (not to the separate fractal cluster) and are normalized to three-dimensional volume. Thus, all the information about fractality remains in the velocity field properties (power-law behavior, parameters h and ξ). The calculation of $\bar{G}(\vec{r}-\vec{r}\,',t)$ is conducted by means of the "cross" diagrammatic technique developed in Ref. [13] and applied in Refs. [14–16] for transport phenomena in disordered media.

Taking into account Eqs. (2) and (10), we rewrite Eq. (9) in Laplace representation

$$\left\{p + u_i\frac{\partial}{\partial x_i}\right\}G(\vec{r},\vec{r}\,',p) = \delta(\vec{r}-\vec{r}\,') + \hat{T}(\vec{r})G(\vec{r},\vec{r}\,',p), \tag{11}$$

where

$$\hat{T}(\vec{r}) = -V_i\frac{\partial}{\partial x_i}. \tag{12}$$

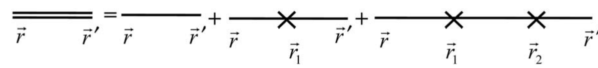
Considering $\hat{T}(\vec{r})$ as the perturbation operator we represent a solution of Eq. (11) in the form of a successive approximation series

$$G(\vec{r},\vec{r}\,',p) = G_0(\vec{r}-\vec{r}\,',p) + \int d^3r_1 G_0(\vec{r}-\vec{r}_1,p)T(\vec{r}_1) \times G_0(\vec{r}_1-\vec{r}\,',p) + \int\int d^3r_1 d^3r_2 G_0(\vec{r}-\vec{r}_1,p)\hat{T}(\vec{r}_1) \times G_0(\vec{r}_1-\vec{r}_2,p)\hat{T}(\vec{r}_2)G_0(\vec{r}_2-\vec{r}\,',p) + \cdots, \tag{13}$$

where G_0 is the unperturbed Green's function obeying the equation

$$\left\{p + u_i\frac{\partial}{\partial x_i}\right\}G_0(\vec{r}-\vec{r}\,',p) = \delta(\vec{r}-\vec{r}\,').$$

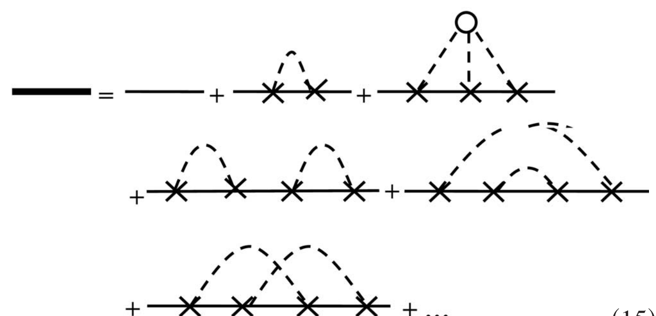
The following diagrammatic representation corresponds to the analytical series (13)



(14)

Here, the double line stands for the function $G(\vec{r},\vec{r}\,',p)$, each thin line for $G_0(\vec{r}_n-\vec{r}_m,p)$, the arguments $\vec{r}_n$ and $\vec{r}_m$ correspond to the endpoints of G-lines, crosses stand for perturbation operator (12). The integration over all inner arguments is implied.

Now we need to average the series of Eq. (14) over the ensemble of medium realization. Velocity fluctuations appearing in the cross elements of Eq. (14) are the subject to average. As usual, the averaged product of an arbitrary number of random factors is reduced to a sum of products of irreducible averages (cumulants). Each term of the sum is associated with a certain partitioning of the initial product into groups of the factors. The result of the averaging of Eq. (14) takes the form



(15)

Here the thick line represents the averaged Green's function $\bar{G}$ and dashed lines connect crosses belonging to a common cumulant. Further, we pick out all strongly connected diagrams (which cannot be divided into two parts by cutting of single G_0-line). Denoting the sum of these irreducible diagrams as $\hat{M}$, we can represent the expansion (15) in the form

$$\bar{G} = G_0 + G_0\hat{M}G_0 + G_0\hat{M}G_0\hat{M}G_0 + \ \ldots\ .$$

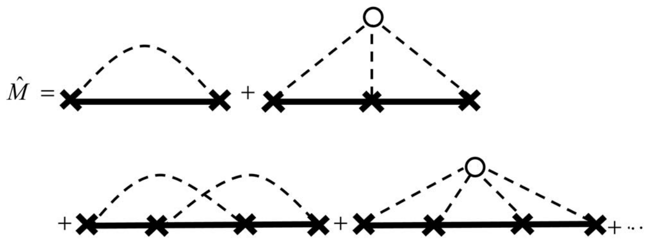
This series is equivalent to the equation

$$\bar{G} = G_0 + G_0\hat{M}\bar{G},$$

having the following analytical representation

$$\bar{G}(\vec{r}-\vec{r}\,',p) = G_0(\vec{r}-\vec{r}\,',p) + \int d^3r_1 d^3r_2 G_0(\vec{r}-\vec{r}_1,p) \times M(\vec{r}_1-\vec{r}_2,p)G_0(\vec{r}_1-\vec{r}\,',p). \tag{16}$$

The regrouping of the terms in diagram expansion for $\hat{M}$ allows one to represent this expansion as the sum of irreducible skeleton diagrams [17]



(17)

Here all the solid lines correspond to $\bar{G}$-functions. In Fourier representation, Eq. (16) takes the form

$$\bar{G}\{\vec{k},p\} = \frac{1}{p + i\vec{k}\cdot\vec{u} - M\{\vec{k},p\}}. \tag{18}$$

In this representation of Eq. (17), gradients of perturbation operator (12) are replaced by the wave vectors multiplied by i, corresponding to the arguments of adjoining horizontal lines (either of two by virtue of incompressibility condition). Each dashed line arising from a cross is associated with its own wave vector over which integration is performed. For every cross vertex (as well as for every "source" of dashed lines connecting the crosses of a particular cumulant), the conservation law for wave vectors is fulfilled. Substituting Eq. (18) into diagrammatic expansion (17), one obtains an integral equation for $M\{\vec{k},p\}$.

This technique was originally intended [13] (see also Ref. [17]) for the calculation of electron's Green function in impurity metals. The theory of impurity metals [13,17] uses essential simplifications, connected with small impurity concentration and the proximity of the electron momenta to Fermi momentum. This allowed confining the calculations of $M\{\vec{k},p\}$ with the first skeleton diagram in Eq. (17). Such simplifications are impossible in our problem because all the diagrams in Eq. (17) are of the same order of magnitude. However, correlation functions over which the diagrammatic expansion takes place possess the property of scaling invariance (7). Thus, it is natural to suppose (and then to prove) that the mass operator itself also possesses this property. According to this conjecture, scaling relations for $M\{\vec{k},p\}$ should have the form

$$M\{\lambda^{-1}\vec{k},\lambda^{-\Delta}p,\lambda\xi\} = \lambda^{-\Delta}M\{\vec{k},p,\xi\}. \tag{19}$$

Here we also supposed the equality of the scaling dimensions Δ of Laplace variable and mass operator, which follows from their additive entry into Eq. (18).

Consider an arbitrary summand of the expansion (17) containing, for example, n-point correlation function. Scaling index of this term is the sum of the indexes of the elements of this diagram. These elements include n-point group of velocity correlators [scaling index $n(h-3)$], n gradients (n), $(n-1)$ Green's functions $[(n-1)\Delta_G]$, and $3n$-dimensional differential of wave vectors ($3n$). (Since the wave vectors $\vec{q}$ used as integration variables are combined additively with $\vec{k}$, the corresponding scaling indexes are equal to that of $\vec{k}$.) The sum of the enumerated indexes is equal to the scaling dimension of M, thus we obtain

$$\Delta = n + (n-1)\Delta_G + n(h-3) + 3n. \tag{20}$$

Taking into account that according to definition, scaling dimensions of Fourier-Laplace transform of Green's function Δ_G and Laplace variable Δ are connected with the relation

$$\Delta_G = -\Delta, \tag{21}$$

we arrive at

$$\Delta = 1 + h, \tag{22}$$

independently of the order of diagram. Since Eq. (19) with $\Delta = 1+h$ is valid for each term in Eq. (17), it holds for the series as a whole.

From the relations stated we may write a general form of mass operator

$$M\{\vec{k},p\} = -pF(\eta,k\xi), \quad \eta = k^2\left(\frac{Va^h}{p}\right)^{2/(1+h)}, \tag{23}$$

where F is the dimensionless function of the two dimensionless variables and the factor Va^h is obtained by using Eq. (4). One more conclusion concerns the mean velocity $\vec{u}$. Both the advection velocity $\vec{V}(r)$ and $\vec{u}$ have the same physical dimensions and so should have the same scaling dimensions. Hence, we have

$$\Delta_u = \Delta_V = h. \tag{24}$$

Thus, as soon as the mean velocity may depend only on correlation length, we may write

$$u \sim V\left(\frac{a}{\xi}\right)^h. \tag{25}$$

Note that $u=0$ in the limit $\xi \to \infty$.

IV. MASS OPERATOR ASYMPTOTICS

Let us begin with the analysis of the expression for the first diagram in Eq. (17)

$$M\{\vec{k},p\} \approx -k_i k_j \int \frac{K_{ij}^{(2)}\{\vec{q},\xi\}d^3q}{p + i\vec{k}\cdot\vec{u} - i\vec{q}\cdot\vec{u} - M\{\vec{k}-\vec{q},p\}}. \tag{26}$$

In this section, we assume Laplace variable p to be real and positive, while in the next section p takes arbitrary complex values, and there $M\{\vec{k},p\}$ becomes the analytic continuation from the real positive semiaxis into the whole of the complex plane. Two limiting cases are possible. The first one corresponds to the inequality

$$\max\left\{k,\left(\frac{p}{Va^h}\right)^{1/(1+h)}\right\} \geqslant \xi^{-1}. \tag{27}$$

In the limit $\xi \to \infty$, the mass operator becomes equal to that obtained in Ref. [9]. It follows from the results of Ref. [9] that the integral (26) (at $\xi \to \infty$) is dominated by

$$q \geqslant k \text{ when } p \leqslant Va^h k^{1+h} \text{ and } q \geqslant (p/Va^h)^{1/(1+h)} \text{ when } p \geqslant Va^h k^{1+h}. \tag{28}$$

Hence in the case of finite value of ξ, obeying the inequality

(27), the function $K_{ij}^{(2)}\{\vec{q},\xi\}$ in the integrand of Eq. (26) can be replaced by the expression given by the first line of Eq. (6) and the terms $i\vec{q}\cdot\vec{u}$ and $i\vec{k}\cdot\vec{u}$ in denominator of Eq. (26) may be neglected:

$$M\{\vec{k}-\vec{q},p\} \sim Va^h q^{1+h} \gg qu,ku.$$

This remains valid for all diagrams of the higher order of the diagrammatic expansion (17). As a result, an expression for $M\{\vec{k},p\}$ in the limit (27) takes the form

$$M\{\vec{k},p\} = -pF(\eta,k\xi) \approx -pF(\eta,\infty) = -p\varphi(\eta), \quad (29)$$

where the properties of function $\varphi(\eta)$ are described in Ref. [9].

In the opposite limiting case,

$$\max\left\{k,\left(\frac{p}{Va^h}\right)^{1/(1+h)}\right\} \ll \xi^{-1}, \quad (30)$$

the main contribution to the integral (26) is given by the values of q of the order of ξ^{-1}. Therefore, in the denominator of Eq. (26), we can neglect both p and $i\vec{k}\cdot\vec{u}$ and put $M\{\vec{k}-\vec{q},p\} \cong M\{-\vec{q},0\} \sim Va^h q^{1+h}$. As a result, the integral does not depend on the variables p and $\vec{k}$ and is of the order of $u\xi$. The same is valid for all the integrals of higher-order diagrams in the expansion (17). Accordingly, the mass operator takes the form

$$M\{\vec{k},p\} \approx -Dk^2, \quad (31)$$

where, in accordance with Eq. (25), the effective diffusion coefficient is

$$D \sim u\xi. \quad (32)$$

V. TRACER CONCENTRATION BEHAVIOR

Now we proceed to analyze the tracer concentration behavior at times great enough for the size of tracer cloud to essentially exceed its initial value. In this case, Green's function directly describes the concentration behavior. The function $\bar{G}(\vec{r},t)$ is determined by inverse Fourier-Laplace transformation of the function (18):

$$\bar{G}(\vec{r},t) = \int_{b-i\infty}^{b+i\infty} \frac{dp}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\vec{k}\cdot\vec{r}+pt)}{p+i\vec{k}\cdot\vec{u}-M(\vec{k},p)}, \quad \mathrm{Re}\, b > 0. \quad (33)$$

The analysis shows that the behavior of the system is principally different in two time intervals with the boundary between them determined by

$$t_* = \frac{\xi}{u} \approx \frac{\xi^{1+h}}{Va^h}. \quad (34)$$

In the interval $t \ll t_*$, for the main body of particles' cloud the Laplace variable in the integrand of Eq. (33) obeys the condition $p \sim t^{-1} \gg t_*^{-1} \approx Va^h \xi^{-(1+h)}$. Thus, the inequality (27) is fulfilled and the mass operator is determined by the expression (29). In this case characteristic values of k in Eq. (33) are of the order of $k \sim (p/Va^h)^{1/(1+h)}$. From here, the inequality $ku \ll p,M$ follows and hence we can neglect the term $i\vec{k}\cdot\vec{u}$ in the denominator of Eq. (33). This is all the more true in the region of tails as far as there $p \gg t^{-1}$ is valid. Thus in the interval $t \ll t_*$, we come to the superdiffusive regime studied in [9]. In this regime the size of the particles' cloud $\sim R(t)$, which grows with time according to

$$R(t) = (Va^h t)^{1/(1+h)} \quad (35)$$

(the formula (23) of the work [9]) and the Green's function at long distances behaves as

$$\bar{G}(\vec{r},t) \cong \frac{B}{(4\pi)^{3/2} R(t)^3} \zeta^{3(1-h)/(1+h)} \exp\{-C\zeta^{1+h/h}\}, \quad \zeta = \frac{r}{R(t)}, \quad (36)$$

where $R(t)$ is determined by Eq. (35) and the constants $B,C \sim 1$ were calculated in Ref. [9].

Note that the relation between the exponent $\frac{1}{1+h}$ in Eq. (35) and the exponent $\frac{1+h}{h}$ of the scaled variable ζ in Eq. (36) is consistent with the relation between fractal dimension of random walk d_w^{-1} and the exponent $u = \frac{d_w}{d_w-1}$ of scaled variable for subdiffusion on fractal (see, e.g., Ref. [11]). These relations predict that in superdiffusive regime the concentration at long distances decays faster than a Gaussian, while in subdiffusive mode the situation is inverse one.

At large times $t \gg t_*$, for the main body of particles' cloud we have an estimate $p \sim t^{-1} \ll t_*^{-1}$. Hence, it follows the inequality $p \ll Va^h \xi^{-(1+h)}$ corresponding to the condition (30). Therefore, the mass operator takes the form (31) and the integral in Eq. (33) leads to the classical diffusion expression

$$\bar{G}(\vec{r},t) \approx (4\pi Dt)^{-3/2} \exp\left(-\frac{(\vec{r}-\vec{u}t)^2}{4Dt}\right). \quad (37)$$

This result is in agreement with the one proposed in Ref. [3], where tracer dispersion in fractal media with finite correlation length has been studied numerically.

Now consider the far region of concentration tail. As it follows from the calculations, the characteristic wave vector values dominating the integral in Eq. (33) are determined by

$$k \sim \max\left\{\frac{1}{\sqrt{Dt}},\frac{r}{2Dt}\right\} \quad (38)$$

and for distances

$$r \gg ut \quad (39)$$

the condition (30) violates. Therefore, at distances of Eq. (39), the calculation in Eq. (33) must be performed using the mass operator determined by Eq. (29). This leads to a superdiffusive form (36) of the concentration tail at $r \gg ut$. As a result, the tail at $t \gg t_*$ has a two-stage structure with the near part determined by the classical diffusion law of Eq. (37) and the remote part determined by the superdiffusive expression of Eq. (36). Comparing Eqs. (36) and (37), one can see that the remote superdiffusive part of the tail decays faster than the near part.

The transition from one tail profile to another takes place at $r_* \sim ut$. A characteristic value of the concentration there has an estimate

$$c(r_*,t) \propto \exp\left(-A\frac{t}{t_*}\right), \tag{40}$$

where $A \sim 1$.

Note that the time t_* [Eq. (34)] of the transition from superdiffusive regime to classical diffusion may be interpreted in two different ways. On the one hand, at times $t \sim t_* = \frac{\xi^{1+h}}{Va^h}$ the particles' cloud size becomes of the order of correlation length $R(t_*) \sim \xi$. So, the medium may be considered as statistically homogeneous at $t > t_*$ with tracer particle dispersion varying according to the classical diffusion law (37). On the other hand, from the estimate $R(t_*) \sim \xi$, we come to the relations

$$\frac{R(t_*)}{u} \sim \frac{\xi}{u} \sim \frac{\xi^{1+h}}{Va^h} \sim \frac{\xi^2}{D}, \tag{41}$$

which mean that the particles' cloud size determined either by superdiffusive or classical diffusive regime at $t \sim t_*$ becomes comparable with the displacement ut_* due to the drift with the mean velocity u.

A qualitative illustration concerning the formation of concentration tails can be made. At early times $t < t_*$, tracer transport occurs with velocities $v(r) \sim V\left(\frac{a}{r}\right)^h$. At large times $t > t_*$, the majority of tracer particles move with the velocities $v \sim v(\xi)$ uncorrelated with respect to directions at the distances $r > \xi$. As usual, this leads to the classical diffusion. One can say that "collisions" of moisture elements destroy the correlations at $r > \xi$. However, some particles exist retaining their correlated motion (without "collisions"). As a result, these particles cover much more distance than the particles in the main body. Their number is small and they form superdiffusive tail at far distances at large times. So, the region of superdiffusive tail at $t > t_*$ is determined by the condition $r > v(\xi)t$. In this region, classical diffusion with the length of a jump ξ and the velocity $v(\xi)$ at each jump is impossible.

VI. CONCLUSION

In summary, the influence of a finite correlation length on the process of tracer random advection in fractal medium has been investigated. A characteristic time t_* is found, which separates the intervals of different transport regimes. At early times $t \ll t_*$, the transport goes into superdiffusive regime studied previously in Ref. [9]. At late times $t \gg t_*$ (when the particles' cloud size becomes greater than the correlation length), this regime transforms to classical diffusion, with the effective diffusion coefficient determined by the product of drift velocity and correlation length. At these times, however, the classical-diffusion profile of concentration holds only in the main body and the near part of concentration tail. Outside this region (in the "far tail"), the concentration profile is described by superdiffusive asymptotics.

A two-stage structure of the tail was obtained earlier in another model (see Ref. [19]) and we suppose it is a general property of systems, in which the type of transport regime changes with time. In these cases, the more distant part of the tail is considered, the earlier transport regime determines its structure.

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[1] M. B. Isichenko, Rev. Mod. Phys. **64**, 961 (1992).
[2] M. Sahimi, Rev. Mod. Phys. **65**, 1393 (1993).
[3] R. A. Guyer, Phys. Rev. B **34**, 7816 (1986).
[4] A. Compte and M. O. Cáceres, Phys. Rev. Lett. **81**, 3140 (1998).
[5] K. Chukbar and V. Zaburdaev, Phys. Rev. E **71**, 061105 (2005).
[6] E. Bonnet, O. Bour, N. E. Odlimg, P. Davy, I. Main, P. Cowie, and B. Berkowitz, Rev. Geophys. **39**, 347 (2001).
[7] M. Sahimi, M. C. Robertson, and C. G. Sammis, Phys. Rev. Lett. **70**, 2186 (1993).
[8] D. L. Koch and J. F. Brady, Phys. Fluids A **1**, 47 (1989).
[9] A. M. Dykhne, I. L. Dranikov, P. S. Kondratenko, and L. V. Matveev, Phys. Rev. E **72**, 061104 (2005).
[10] R. A. Guyer, Phys. Rev. A **32**, 2324 (1985).
[11] H. E. Roman and M. Giona, J. Phys. A **25**, 2107 (1992).
[12] D. ben-Avraham and S. Havlin, *Diffusion and Reaction in Fractals and Disordered Systems* (Cambridge University Press, Cambridge, 2000).
[13] A. A. Abrikosov and L. P. Gor'kov, Sov. Phys. JETP **8**, 1090 (1958); **9**, 220 (1959).
[14] V. M. Finkel'berg, Sov. Phys. JETP **26**, 268 (1967).
[15] Yu. A. Dreizin and A. M. Dykhne, Sov. Phys. JETP **36**, 127 (1972).
[16] J.-P. Bouchaud and A. Georges, Phys. Rep. **195**, 127 (1990).
[17] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Prentice Hall, Englewood Cliffs, NJ, 1963).
[18] S.-K. Ma, *Modern Theory of Critical Phenomena* (Benjamin, London, 1976).
[19] A. M. Dykhne, P. S. Kondratenko, and L. V. Matveev, JETP Lett. **80**, 410 (2004).